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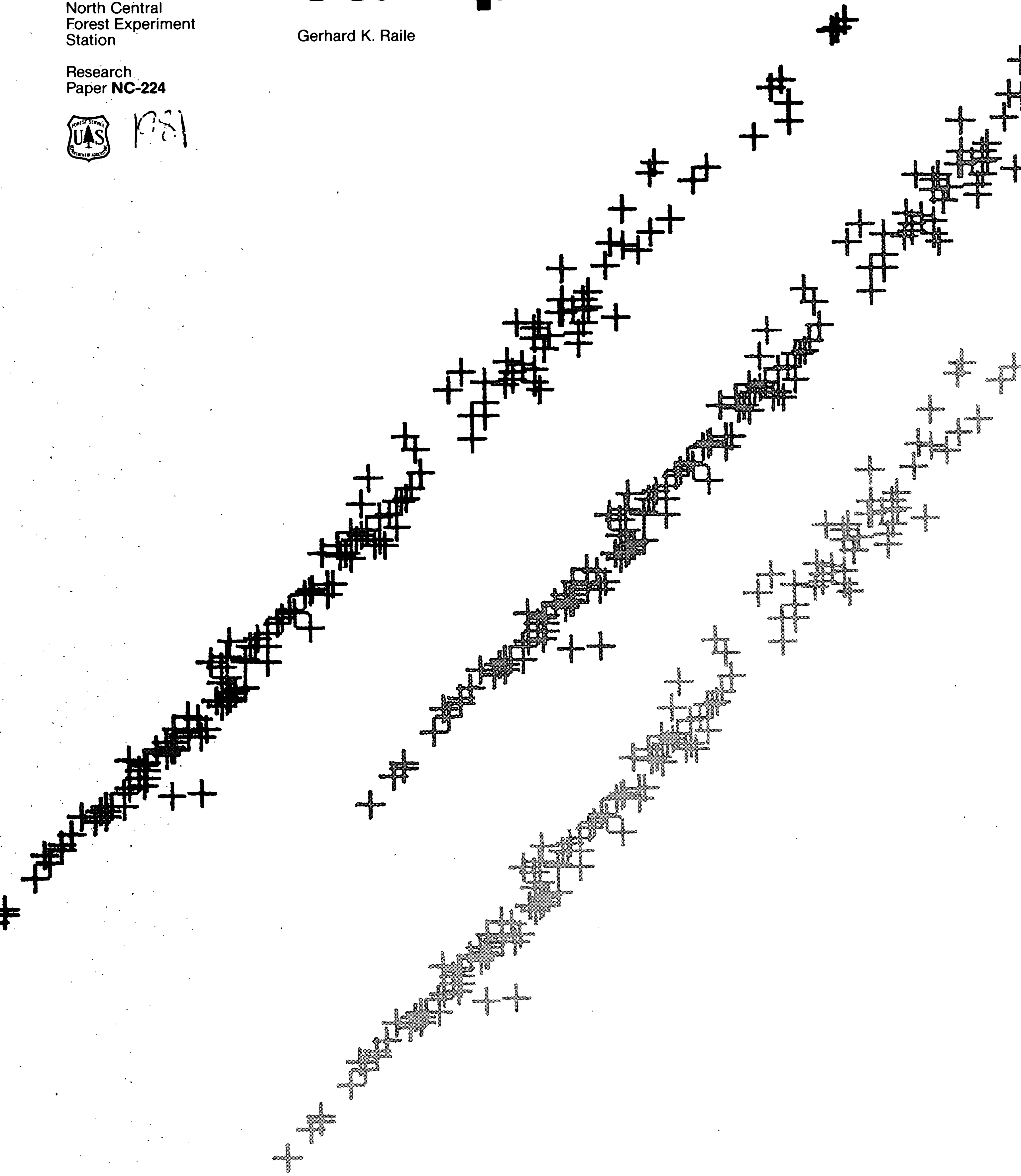

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ESTIMATING STUMP VOLUME

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A technique for estimating volume in stumps[1] of various heights given d.b.h. (tree diameter at breast height), has several uses: (1) stump volumes can be used as one component in calculating total above-ground biomass; (2) tree utilization factors can be computed; (3) an economic analysis can be made of harvesting to various stump heights; and (4) forest inventory statistics, which assume a known stump height, can be adjusted for any other stump height.[2]

Interest in stump weight and volume has developed recently as foresters have sought ways to produce and harvest more usable tree biomass per acre of forest land. Young *et al.* (1964) were among the first to develop regression equations to estimate the weight of the complete tree and its components. Young and other researchers have produced regression equations that use d.b.h. to predict the combined weight of stump and/or roots (King and Schnell 1972, Schnell and Toennisson 1978, Dyer 1967, Keays 1971).

Few studies deal with stump volume (as opposed to weight) as a separate component of total volume. Drolet (1973) presents data on the volume of merchantable wood left in stumps after normal harvesting. Decei *et al.* (1975) developed a linear regression model for spruce to predict stump volume from the bole volume. They found the average ratio of stump volume to bole volume to be 0.101 under normal harvesting conditions.

[1]*Stump is defined as the tree bole from ground level to any height less than or equal to 4.5 feet.*

[2]*For applications where d.b.h. is unknown, equations that predict d.b.h. from stump measurements may be used (Raile 1978).*

OBJECTIVE AND MODEL CRITERIA

The objective of this study was to develop a technique for estimating the volume both inside and outside bark of any stump section between ground level and 4.5 feet given d.b.h. and stump heights (h). Equations were developed that predict stump diameter inside (d.i.b.) and outside (d.o.b.) bark. These equations are used to compute stump volume both with and without bark.

These models had to: (1) fit the data well, (2) give reasonable diameter estimates between ground level and 4.5 feet, (3) give consistent results for d.i.b. and d.o.b., (4) use independent variables normally available in existing inventory data, and (5) be functions that can be modified and/or integrated to compute stump volume.

The following mathematical technique is used in computing volume:

Given that d = f (d.b.h., h),
where h = height above ground level ($0 \leq h \leq 4.5$),
d.b.h. = a constant, and
d = either d.o.b. or d.i.b. at height h;

then the volume in the stump section (V) between heights a and b is

$$V = \frac{\pi}{4}\int_a^b [\, f(\text{d.b.h.}, h) \,]^2 \, dh, \quad 0 \leq a < b \leq 4.5 \qquad (1)$$

DATA

Data were collected from 2,975 trees as part of forest product utilization studies conducted in conjunction with forest inventories in Michigan, Minnesota, and Wisconsin (tables 1 and 2). Measurements taken from random samples of felled trees at logging operations in these States were d.b.h., double bark thickness at d.b.h., stump height, and d.i.b. at stump height. In addition, the d.o.b. was measured at half-foot intervals from 0.5 to 2.5 feet above ground level. On slopes, measurements were taken from ground line on the uphill side. All measurements were taken to the last 0.1 inch. Where an abnormality such as a bulge or a fork occurred, the observation was not taken.

MODEL FOR PREDICTING D.O.B.

Pearson correlation coefficients and stepwise regression techniques were used to develop the following model:

$$\text{d.o.b.} = \text{d.b.h.} + B(\text{d.b.h.})\frac{4.5-h}{h+1}, \qquad (2)$$

where B is the species group regression parameter.

This model was fit for 22 species groups. The species included in each species group are listed below. Table 1 presents the regression coefficient, coefficient of determination (R^2) and standard error (SE) for each species group. Figure 1 shows how the model fits the data. In this graph the predicted d.o.b. is plotted against a 10-percent random sample of observed d.o.b. for red pine, *Pinus resinosa* Lamb.

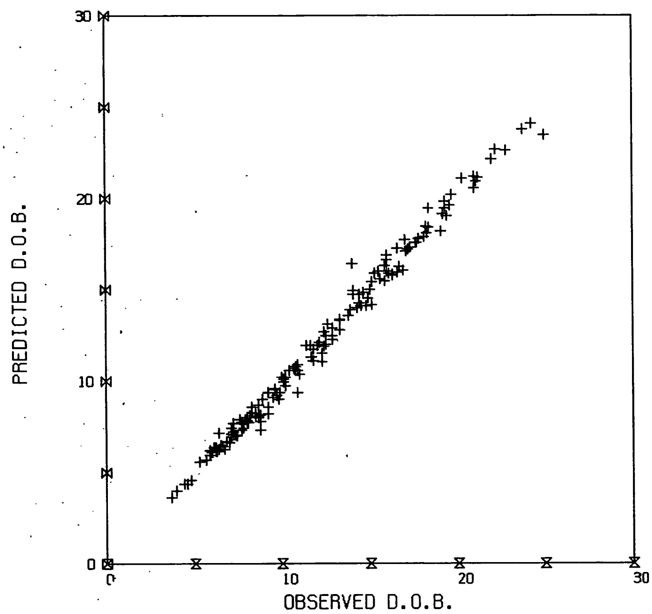

Figure 1.—*Predicted vs. observed d.o.b. for red pine.*

SPECIES GROUP LIST

Common name	**Scientific name**
SOFTWOODS	
Eastern white pine.	*Pinus strobus* L.
Red pine.	*Pinus resinosa* Ait.
Jack pine.	*Pinus banksiana* Lamb.
White spruce.	*Picea glauca* (Moench) Voss
Black spruce.	*Picea mariana* (Mill.) B.S.P.
Balsam fir.	*Abies balsamea* var. *balsamea* (L.) Mill.
Hemlock.	*Tsuga canadensis* (L.) Carr.
Northern white-cedar. . .	*Thuja occidentalis* L.
HARDWOODS	
White oaks.	*Quercus alba* L.
	Quercus bicolor Willd.
	Quercus macrocarpa Michx.
Red oaks.	*Quercus rubra* L.
	Quercus ellipsoidalis E.J. Hill
Beech.	*Fagus grandifolia* Ehrh.
Yellow birch.	*Betula alleghaniensis* Britton
Hard maples.	*Acer nigrum* Michx.f.
	Acer saccharum Marsh.
Soft maples.	*Acer rubrum* var. *rubrum* L.
	Acer saccharinum L.
White & green ash.	*Fraxinus americana* L.
	Fraxinus pennsylvanica Marsh.
Black ash.	*Fraxinus nigra* Marsh.
Paper birch.	*Betula papyrifera* var. *papyrifera* Marsh.
Bigtooth aspen.	*Populus grandidentata* Michx.
Quaking aspen.	*Populus tremuloides* Michx.
Basswood.	*Tilia americana* L.
Cottonwood.	*Populus deltoides* Bartr. ex Marsh.
Elms.	*Ulmus americana* L.
	Ulmus rubra Muhl.
	Ulmus thomassii Sarg.

MODEL FOR PREDICTING D.I.B.

To estimate d.i.b., equation (2) was modified as follows:

$$\text{d.i.b.} = A(\text{d.b.h.}) + B(\text{d.b.h.})\frac{4.5-h}{h+1} \qquad (3)$$

where A and B are the species group regression parameters (table 2).

D.i.b. was only measured at stump height and 4.5 feet. To estimate d.i.b. at half-foot intervals from 0.5 to 2.5 feet, it was assumed that the ratio of d.i.b. to d.o.b. at stump height is constant in this range. This assumption increased the number of observations available for fitting the equations.

Table 1.—*Stump d.o.b. regression coefficients for tree species of the Lake States*

Species group	Trees	D.b.h. range	Observations	D.o.b. range	Species parameter B	R^2	SE
	No.	*Inches*	*No.*	*Inches*			*Inches*
E. white pine	53	6.2-33.0	359	6.2-47.2	0.11694	0.89	1.2
Red pine	228	3.4-23.0	1,544	3.4-27.9	.08091	.91	.5
Jack pine	579	3.4-19.4	3,843	3.4-24.0	.08076	.87	.5
White spruce	34	5.1-18.0	223	5.1-30.9	.16903	.86	1.2
Black spruce	103	3.6-17.9	712	3.6-27.4	.12147	.73	.9
Balsam fir	119	4.3-15.4	706	4.3-26.0	.15359	.89	.8
Hemlock	57	5.8-29.0	371	5.8-42.5	.12667	.85	1.3
N. white-cedar	14	4.8-13.3	96	4.8-22.3	.18850	.89	.9
White oaks	61	4.2-26.2	377	4.2-43.7	.14872	.84	1.3
Red oaks	214	2.5-28.7	1,309	2.5-37.9	.12798	.83	1.2
Beech	29	4.5-24.3	167	4.5-38.8	.15113	.79	1.8
Yellow birch	41	7.5-28.1	271	7.5-39.4	.15350	.78	2.0
Hard maples	132	2.3-31.3	743	2.3-37.8	.12111	.76	1.6
Soft maples	74	2.5-20.8	340	2.5-28.0	.11585	.77	1.2
White & green ash	37	7.3-24.7	256	7.3-44.8	.12766	.75	1.5
Black ash	15	7.9-17.5	87	7.9-25.5	.17376	.93	.9
Paper birch	178	3.2-22.4	974	3.2-34.7	.11655	.77	1.0
Bigtooth aspen	204	4.0-15.6	895	4.0-21.6	.06834	.82	.5
Quaking aspen	678	2.9-20.5	3,920	2.9-29.0	.09658	.83	.8
Basswood	38	6.4-26.7	178	6.4-42.0	.14413	.86	1.4
Cottonwood	7	12.8-27.8	42	12.8-48.2	.17123	.85	2.1
Elms	80	7.0-30.5	464	7.0-49.2	.16638	.84	1.6

Table 2.—*Stump d.i.b. regression coefficients for tree species of the Lake States*[1]

Species group	Observations	D.I.B. range	Species parameter A	Species parameter B	R^2	SE
	No.	*Inches*				*Inches*
E. white pine	360	6.0-43.9	0.91385	0.11182	0.86	1.2
Red pine	1,551	3.0-25.8	.90698	.08469	.87	.7
Jack pine	3,873	3.1-23.4	.90973	.07926	.84	.6
White spruce	225	4.9-29.7	.95487	.15664	.83	1.2
Black spruce	713	3.4-26.4	.94122	.11781	.69	1.0
Balsam fir	725	4.0-25.4	.93793	.14553	.87	.9
Hemlock	375	5.0-41.0	.91400	.11975	.79	1.4
N. white-cedar	96	4.6-22.2	.94698	.18702	.86	1.0
White oaks	384	3.7-42.3	.91130	.14907	.83	1.4
Red oaks	1,339	2.3-40.3	.92267	.12506	.81	1.3
Beech	172	4.3-36.5	.96731	.14082	.79	1.6
Yellow birch	273	7.1-37.8	.94423	.14335	.80	1.7
Hard maples	771	2.1-36.7	.93818	.11424	.75	1.5
Soft maples	369	2.4-26.3	.94181	.10740	.73	1.2
White & green ash	256	6.8-41.8	.91979	.12152	.72	1.6
Black ash	90	7.6-24.1	.93502	.17071	.94	.8
Paper birch	1,016	3.0-32.1	.93763	.10640	.75	.9
Bigtooth aspen	981	3.7-19.7	.91625	.06478	.71	.7
Quaking aspen	4,046	2.6-26.5	.91882	.08593	.78	.8
Basswood	192	6.1-37.3	.92442	.14240	.87	1.3
Cottonwood	43	11.8-47.1	.92736	.17626	.85	2.2
Elms	477	6.4-43.8	.93257	.15803	.82	1.6

[1]Number of trees and d.b.h. ranges for these regressions are the same as those given in Table 1.

COMPUTING STUMP VOLUME

The volume inside bark (V_i) for a section of the stump between a and b can now be found by substituting equation (3) into equation (1) and evaluating the definite integral. Because d.b.h. is in inches and h is in feet, the equation to compute the volume in cubic feet becomes:

$$V_i = \frac{\pi}{4(144)} \int_a^b \left[A(d.b.h.) + B(d.b.h.) \frac{4.5-h}{h+1} \right]^2 dh \quad (4)$$

Integration of equation (4) gives:

$$V_i = \frac{\pi(d.b.h.)^2}{4(144)}$$

$$\left[(A-B)^2 h + 11B(A-B) \ln(h+1) - \frac{30.25}{h+1} B^2 \right]_a^b \quad (5)$$

The solution for volume outside bark is a special case of equation (5) where parameter A is equal to 1. Therefore equation (5) can be used to estimate stump section volumes both inside and outside the bark.

With this method, volumes both inside and outside bark can be computed for any stump section between ground level and 4.5 feet. The equation can be solved easily using a hand-held, programmable calculator or automated data processing systems. For example, the equation for cubic foot volume outside the bark of a 1-foot red-pine stump becomes:

$$V = 0.008240(d.b.h.)^2$$

where d.b.h. is in inches.

The volumes obtained with this method have several characteristics worth noting. First, with these coefficients, the predicted d.o.b. is always greater than d.i.b. for stump heights between 0.0 and 4.5 feet. Therefore, the volume outside the bark is always greater than the volume inside the bark for any stump section. In addition, the volume obtained from equation (5) is always less than or equal to the volume computed assuming a neiloid frustum.

If direct observations of stump volume and d.b.h. were available, more reliable volume estimates might be obtained by fitting a regression to directly predict volumes. Our results suggest investigating a model as such:

$$V = d.b.h.^2 \left[A + Bh + C[\ln(h+1)] + D \frac{1}{h+1} \right]$$

where A through D are the regression coefficients.

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Equations are presented that predict diameter inside and outside bark for tree boles below 4.5 feet given d.b.h. These equations are modified and integrated to estimate stump volume. Parameters are presented for 22 Lake States species groups.

KEY WORDS: Taper model, Minnesota, Wisconsin, Michigan, biomass.

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