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Calculating Competition In Thinned Northern Hardwoods

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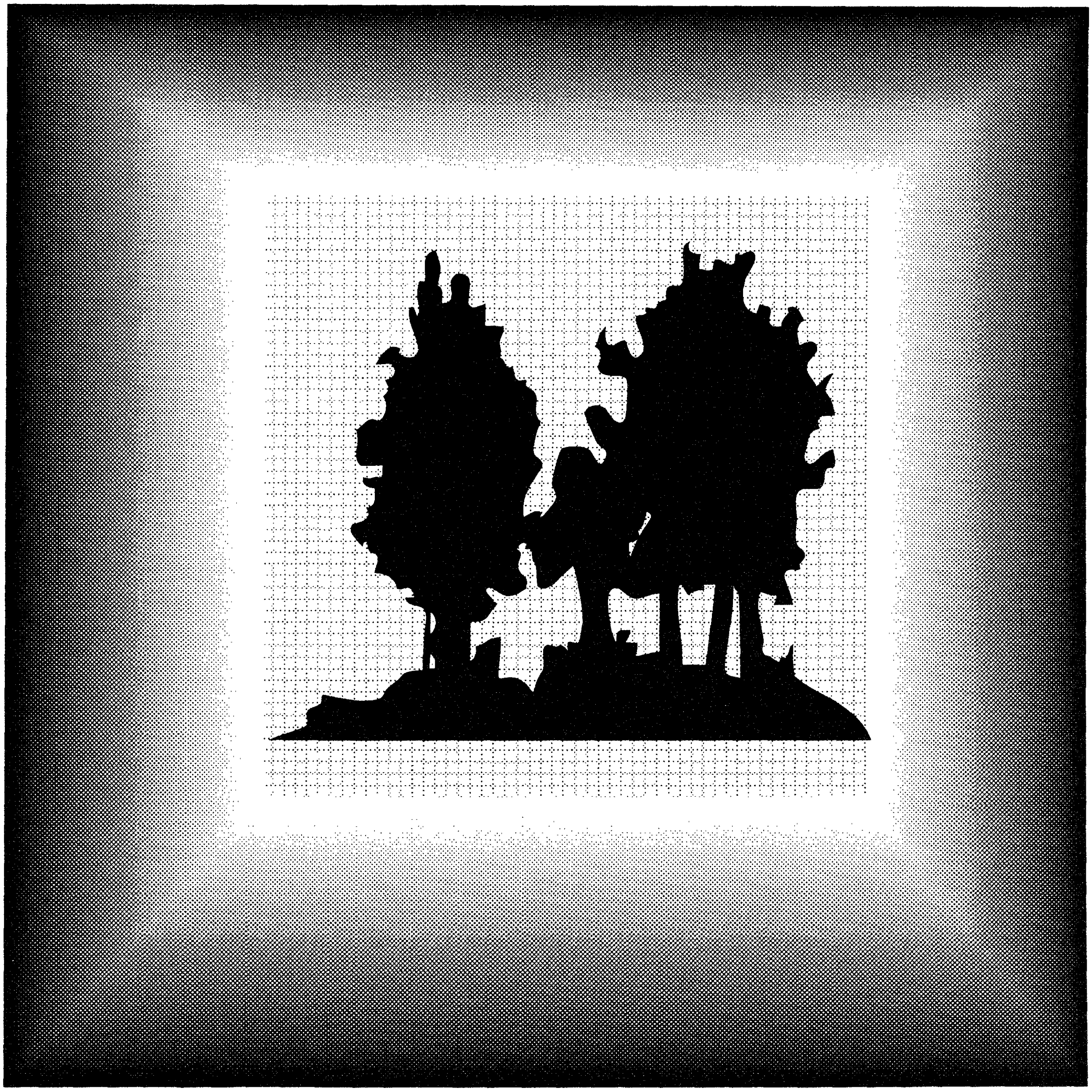

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Diameter at breast height (d.b.h.) and basal area (BA) growth of individual trees in a stand are strongly affected by the amount of competition trees face for light, water, and nutrients. Studies have shown that selected dominant and codominant northern hardwood crop trees are very responsive to crown thinning, especially in the seedling-, sapling-, and pole-size classes (Erdmann 1983). However, forest land managers and researchers must be able to evaluate and compare the effects of alternative thinning treatments and methods of release from competition to ensure the best growth of individual crop trees as well as the entire stand. A competition index that is strongly correlated with future growth would be valuable to evaluate silvicultural thinning practices and to estimate stand growth.

Many competition indexes have been developed that show potential for estimating growth. This paper presents four competition measures and compares their effectiveness as growth predictors for individual trees in a northern hardwood stand. These competition measures can be used to measure how effectively various thinning treatments release crop trees.

COMPETITION MEASURES

Competition Index (CI)

Several attempts have been made to obtain objective measures of competition and/or degree of release of individual trees. Most of this work was based on functions of stem or crown diameters. Stiell (1970) reported that crown competition in red pine plantations is concentrated within the areas occupied by the individual crown, while root competition is diffuse and may arise from trees at a great distance. In contrast to red pine, most forest-grown northern hardwoods on level ground have a uniformly distributed root system with an irregular, elliptical shape (Hannah 1972, Tubbs 1977). Average sugar maple root area coincides closely with crown area, while the longest yellow birch roots are frequently well outside the crown perimeters. However, exceptions are found for both species, particularly in slope-grown or leaning trees. Therefore, a CI should give the most weight to trees whose crowns touch or overlap the subject tree's crown, but should also include a component from larger trees at greater distance that could provide root competition. Ellis (1979) assumed that the effects of individual competitors would be additive, and that the magnitude of the competitive effect would be

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directly related to the size of the competing trees and inversely related to their distances away from the subject tree. He defined a CI of:

$$CI = \sum \frac{d^2}{D^2 A^2} \qquad [1]$$

over all trees within a plot of radius equal to four times the crown radius of the subject tree,

where:
d is the d.b.h. (in inches) of the competitor,
D is the d.b.h. (in inches) of the subject tree, and
A is the distance (in feet) between the subject tree and competitor.

After a thinning, Ellis found the percent reduction in the CI accounted for about 40 percent of the variation in d.b.h. growth of sugar maple over 5 years, but only 14 percent of the variation in black cherry. However, calculating the competition over a fixed radius does not take into account the fact that large trees compete over much greater distances than small ones.

Daniels and Burkhart (1975) found a competition index was necessary to develop growth and mortality models for individual trees. They selected an empirical model developed by Hegyi (1974) that is similar to Ellis' model except that it uses linear rather than squared terms:

$$CI = \sum \frac{d}{D\,A} \qquad [2]$$

over the competing trees with d, D, and A defined as above.

The problem still remains of determining which trees actually compete with the subject tree. Under the assumption that a tree's area of influence will increase as the tree grows, Daniels and Burkhart modified Hegyi's index by choosing competitors based on both size and distance. Trees were included in the CI index if they fell within a 10-BA-factor prism held at the subject tree. This modified index was found to have a correlation coefficient with d.b.h. growth in loblolly pine of r = -0.4, while the Hegyi index, with a fixed 10-foot radius, had an r value of -0.2. This modified CI is incorporated in this work.

Area Potentially Available (APA)

Moore *et al.* (1973) modified Brown's (1965) concept of the area potentially available (APA) to a tree as a measure of point density for use as a competition index. Their modified APA index is defined as the area of an irregular polygon constructed around a subject tree. The edges of the polygon are formed by intersecting lines that divide and are perpendicular to lines connecting the subject tree with each of its competitors (fig. 1). This modified APA index is intended to express aerial and root competition by describing the zone of primary influence and growing space for an individual tree as being limited by competition from surrounding trees.

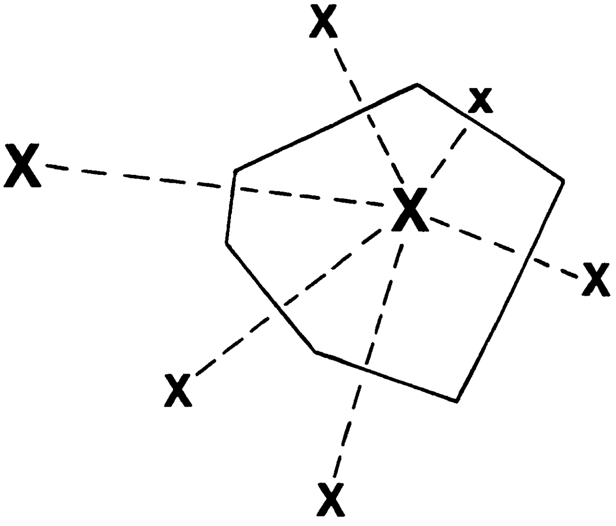
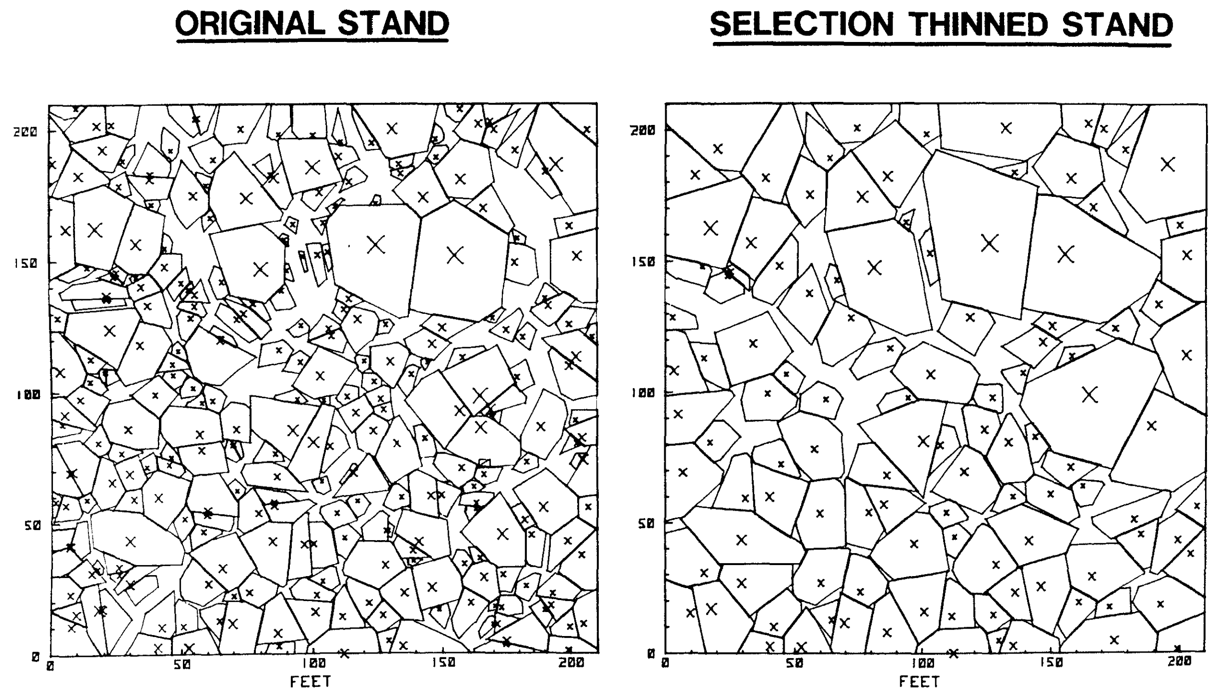


Figure 1.—*APA (Area Potentially Available) polygon.*

Most competition indexes are actually measures of crowding. The APA is unique in that it is a quantitative measure of the competitive status of the tree. The APA has four desirable features:

1. The areas of the trees in a stand are mutually exclusive.
2. The area between two trees is divided relative to tree size.
3. The APA is sensitive to changes in the relative tree sizes over time.
4. Good correlation exists between BA growth and APA number.



ORIGINAL STAND

SELECTION THINNED STAND

Figure 2.—*Mutually exclusive APA polygons for all trees in the stand before and after thinning.*

The area around each tree is influenced by the size and nearness of neighboring trees. In general, the defined polygon of a large subject tree will be restricted only by its nearest neighbors, while a small tree will be affected by larger trees at greater distances. The effect causes "holes" or unused spaces between the polygons where no single tree dominates (fig. 2).

The edges of the polygons divide the line connecting the subject tree with its neighbors. The point of division is based on the relative size of the trees, such as ratio of tree diameters or diameters squared. Pelz (1978) tested division points for both two- and three-dimensional (a geometric solid with a base of the APA polygon and a height equal to the tree height) growing areas. He found high correlation (r = 0.82) between BA growth of yellow poplar and the APA, based on both two and three dimensions with the polygons constructed proportional to d.b.h. squared. Moore *et al.* (1973) also found that APA accounted for much of the variation in BA growth of black oak (r = 0.78), white oak (r = 0.78), and yellow poplar (r = 0.84).

Because the location of the side of the polygon between competing trees is relative to tree size, the APA number is sensitive to changes over time. If one tree grows faster than another, the dividing line between them moves towards the slower growing tree, an indication of their relative vigor and the faster growing tree's ability to command a larger share of the available resources.

APA calculations in this paper use the ratio of d.b.h.^{2} to determine the location of the perpendiculars for constructing the APA polygon; *i.e.*,

$$R = \frac{A D^2}{(D^2 + d^2)} \qquad [3]$$

where:

R = distance to the perpendicular line from subject tree,
A = distance from subject to competing tree (in feet),
D = d.b.h. of the subject tree (in inches), and
d = d.b.h. of the competing tree.

Five trees in each quadrant with perpendiculars closest to the subject tree are determined. From these perpendiculars, the set that gives the simplest convex polygon is selected. The area of this polygon is the APA index for the subject tree.

Crown Release Quotient (CRQ)

The change in the competition indexes before and after thinning should indicate the amount of release an individual tree has received. One competition measure defines a competition circle for each tree and then measures the amount of overlap in these circles. Gerrard (1969) defined a competition quotient (CQ) as:

$$CQ = \frac{1}{A} \sum a \quad \text{over the competing trees} \qquad [4]$$

where:

A is the competition circle area of the subject tree, and

a is the area of overlap of competing tree circles with the competition circle radius defined as a constant times the d.b.h. of the tree.

For the best growth and quality development, pole-sized northern hardwoods need about 7 feet of open growing space around the crown perimeters of crop trees. This amount of crown release can nearly double d.b.h. growth rates to shorten rotations (Erdmann 1986). To measure crown release after a thinning, Gerrard's (1969) competition quotient (CQ) was modified to a crown release quotient (CRQ) by using the crown radius as the competition circle for competing trees, and crown radius + C (where C is the desired clear area) for the subject tree (fig. 3). A value of zero for the CRQ is equivalent to a totally released crown.

Basal Area in 1/20 Acre (BA20)

The basal area of trees within a 26.3-foot radius (1/20 acre) of the subject tree is probably one of the earliest competition measures. It is more a measure of relative density than of true competition and is included because of historic use and ease of computation. The basal areas of all trees within a 26.3-foot radius of, but not including, the subject tree are summed to give the BA20. Although the subject tree is often included in this index, a higher correlation between tree growth and BA20 was found when the subject tree was excluded.

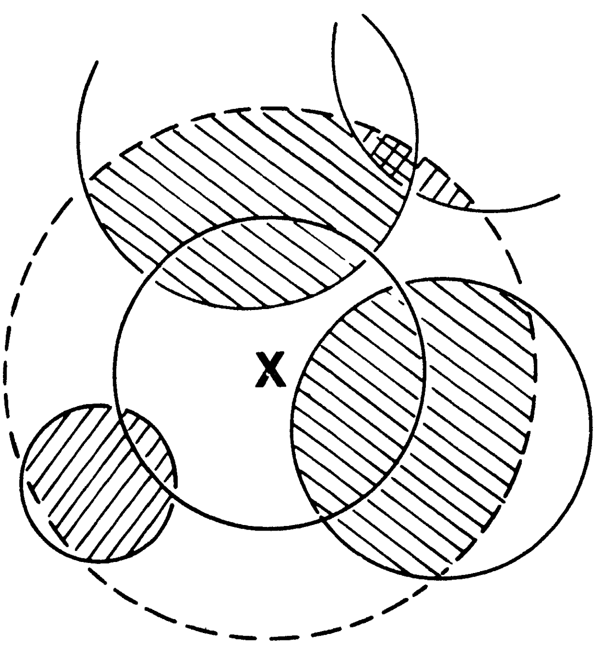


Figure 3.—*Crown release quotient - measuring the overlap between competition circles of the subject tree X (crown radius + desired clear area) and the competing trees (crown radius).*

COMPARING THE COMPETITION MEASURES

A 60-year-old, even-aged, second-growth, pole-size northern hardwood stand in the western Upper Peninsula of Michigan was selected for a mechanized thinning study (Mattson and Winsauer, In prep.). One objective of this study was to determine the effectiveness of the competition indexes in predicting individual tree growth of both the crop trees and all trees after thinning.

The stand is growing on a level, uniform site about 5 miles south of Lake Superior at an elevation of 800 feet. Site index for sugar maple was estimated from stem analysis to be 60 feet, at age 50 years. Before release, the uniformly stocked stand contained 450 trees (2 inches and larger) with a basal area of 132 ft^2/acre. Sugar maple (*Acer saccharum* Marsh.) and red maple (*Acer rubrum* L.), the principal species, accounted for 85 percent and 10 percent of the basal area, respectively. Other species were yellow birch, black cherry, ironwood, elm, and aspen.

Five square, 1-acre plots were established in the stand and all trees 1.55-inch d.b.h. or larger were numbered, and their location, species, and diameter were recorded. Potential crop trees were identified on the basis of species, crown class, tree quality, size, and spacing. Treatments representing possible mechanized harvesting practices were randomly assigned. The five treatments applied were: (1) unthinned control; (2) stand-improvement selection cut to a residual BA/acre of 68 ft^2; (3) strip-only thinning with 10-foot clearcut strips on 26-foot centers; and (4) 10-foot-wide clearcut strips with either 30-foot-wide selection thinning or (5) 70-foot-wide selection thinning between cut strips.

Diameters of trees on these five plots were remeasured after three growing seasons, allowing preliminary comparisons between competition measures, treatments, and growth. The data reported are for the residual trees in the interior 1/2 acre of each plot to avoid error from edge effects. Data for crop trees are reported separately because a major goal of an improvement cut is to increase the growth of the existing quality trees.

There were some major differences between the treatments involving thinning. Most noticeable was the 34-percent loss of crop trees in the strip-only thinning (table 1). For the treatments involving selection thinning, the average stand diameter increased more than 3 inches when all trees were considered, and 1.4 inches for trees 5 inches and larger. There was essentially no change in average stand diameter following strip thinning—only 0.1 inch.

Trees in the four thinned plots had significantly less competition than those in the control plot (table 2). Trees in the strip-thinned plot suffered from more competition than those in the other thinned plots. This was because more small trees were left after that treatment, and all the competition indexes, except BA20, depend upon the d.b.h. of the subject tree. However, when the competition to only crop trees is considered, the strip thinning produced results similar to the stand-improvement selection treatment. The residual BA/acre in the strip-with-selection thinnings was lower than in the other two thinning treatments, reducing the competition on the residual trees in those plots.

Predicting 3-Year Growth Results

The competition indexes after thinning, their change because of treatment, and their percent change were investigated to determine their effectiveness as predictors of growth. No apparent benefit was derived from using either the change or percent change in the indexes, so only the results from the analysis of the indexes are presented. A major objective of the thinnings was to increase the growth of crop trees, so the predictive value of competition measures for crop trees only was also investigated.

Linear correlations between CI or APA and both d.b.h. and BA growth were fairly high ($r = 0.5$ to 0.8) for the three plots with some selective thinning, but were lower in the control and strip-thinned plots, possibly because of the many small, poor-quality trees in those plots. Similar groupings were found in the other index/growth correlations, so results are presented for the correlations pooled over the three plots with some selection thinning, the other two plots (control and strip-only thinnings), and over all plots (table 3). In the plots that had some selection thinning, both CI and APA had higher correlation with d.b.h. growth than did the tree's original d.b.h., a well-known predictor of growth.

Table 1.—*Plot data before and after treatment (full acre)*

Characteristic	**Unthinned control**	**Stand-improv. selection**	**10-foot-wide strip-thinning with selection between**		
			26-foot centers	**40-foot centers**	**80-foot centers**
Stand (all trees 1.55-inch d.b.h. and larger)					
No. of trees					
Before	435	450	487	423	431
After		109	292	82	96
BA/ac (ft^2)					
Before	136	147	121	127	130
After		70	74	56	57
Avg. d.b.h. (inches)					
Before	6.61	6.62	5.94	6.22	6.46
After		10.56	6.02	10.40	9.71
Trees 4.55 inches d.b.h. and larger					
No. of trees					
Before	294	272	290	241	274
After		109	176	81	95
BA/ac (ft^2)					
Before	128	138	110	117	121
After		70	68	55	57
Avg. d.b.h. (inches)					
Before	8.28	9.11	7.87	8.68	8.39
After		10.56	7.97	10.48	9.79
Crop trees					
No. of trees					
Before	60	63	53	57	56
After		63	35	45	45
Avg. d.b.h. (inches)					
Before	9.95	11.13	9.70	11.99	11.41
After		11.13	9.78	12.23	11.41

Table 2.—*Average competition indexes and 3-year tree growth for residual trees (interior 1/2 acre)*

Characteristic	**Unthinned control**	**Stand improv. selection**	**10-foot-wide strip-thinning**		
				with selection between	
			26-foot centers	**40-foot centers**	**80-foot centers**
Stand (trees 4.55 inches d.b.h. and larger)					
APA	100	372	228	518	387
CI	1.45	0.38	0.73	0.31	0.30
CRQ	1.37	0.54	0.69	0.38	0.39
BA20	6.14	3.26	2.71	1.98	2.02
Avg. tree d.b.h. (inches)					
Initial	8.02	10.49	7.90	10.80	9.68
3-year growth	0.23	0.34	0.35	0.37	0.37
Avg. tree BA (ft^2)					
Initial	0.428	0.628	0.384	0.731	0.597
3-year growth	0.025	0.042	0.033	0.049	0.043
Crop trees					
APA	139	431	390	697	574
CI	0.90	0.33	0.35	0.24	0.25
CRQ	1.58	0.51	0.52	0.36	0.41
BA20	7.02	3.10	2.78	2.04	2.01
Avg. tree d.b.h. (inches)					
Initial	10.08	11.19	9.43	12.75	11.86
3-year growth	0.23	0.39	0.44	0.43	0.40
Avg. tree BA (ft^2)					
Initial	0.591	0.701	0.518	0.972	0.887
3-year growth	0.029	0.049	0.047	0.064	0.057

Table 3.—*Linear correlation (r) between tree growth and competition indexes*[1]

	DBH growth			BA growth		
	Plots with selection	**Control & strip**	**All plots**	**Plots with selection**	**Control & strip**	**All plots**
All trees 4.55 inches d.b.h. and larger						
Orig. d.b.h.	0.49	0.54	0.53	0.84	0.88	0.86
APA	0.51	0.45	0.44	0.81	0.59	0.67
CI	-0.61	-0.23	-0.23	-0.67	-0.31	-0.30
CRQ	-0.34	-0.19	-0.21	-0.24	-0.11	-0.13
BA20	-0.32	-0.15	-0.18	-0.31	-0.05	-0.11
Crop trees						
Orig. d.b.h.	0.52	0.55	0.53	0.84	0.79	0.83
APA	0.53	0.31	0.47	0.79	0.57	0.75
CI	-0.58	-0.67	-0.57	-0.65	-0.71	-0.56
CRQ	-0.30	-0.55	-0.39	-0.22	-0.51	-0.28
BA20	-0.38	-0.53	-0.41	-0.38	-0.49	-0.34

[1] Competition indexes as defined in the text.

Table 4.—*Three-year growth prediction equations (variables listed in order of entrance to the model)*

Model	**Independent variables**	r^2
Growth - all trees		
D.b.h. (inches)	= 0.071 + 0.042 d.b.h. - 0.081 CRQ - 0.001 d.b.h.^{2}	0.39
BA (inches^2)	= - 0.0081 + 0.0044 d.b.h. - 0.0079 CRQ + 0.0001 d.b.h.^{2}	0.78
Growth - crop trees		
D.b.h. (inches)	= 0.113 - 0.130 CRQ + 0.041 d.b.h. - 0.001 d.b.h.^{2}	0.48
BA (inches^2)	= - 0.0036 + 0.0030 d.b.h. - 0.0144 CRQ + 0.0002 d.b.h.^{2}	0.76

However, to be of value, the predictive capabilities of an index should be independent of treatment; therefore, data from all treatments were combined for regression analysis. Initial d.b.h. and d.b.h.^{2} are the most commonly used predictors of tree growth; therefore, each competition index was tested to determine if adding these variables would improve the model. Adding a competition index term significantly improved the r^2 value on all equations, especially the models for diameter growth. Adding a CRQ or BA20 term increased the r^2 value of the diameter growth equation by about 20 percent for all trees (from 0.32 to 0.39) and 70 percent when only crop trees (from 0.28 to 0.48) were considered. A CI term was equally effective in the crop tree equations. An APA term did not add as much to the error reduction of the equations because of the high correlation between d.b.h. and APA ($r > 0.7$). Having the d.b.h. term already in the equation explained much of the same error as the APA term.

Other variables, such as age, tree height, site index, crown class, and residual basal area, are also known predictors of growth. Because the growth measurements are from a single, even-age stand, the only additional variable investigated was residual basal area. To determine their effect, the competition indexes were added individually to a model of d.b.h., d.b.h.^{2}, and residual basal area. Adding a CRQ or a BA20 term still significantly improved the r^2 value of the model (a 10-percent increase for d.b.h. growth).

Stepwise regression was used to determine which variable most reduced the error variance (order of entry into the model)—d.b.h., d.b.h.^{2}, residual BA, or the competition indexes. Generally the competition measures entered the model second, after d.b.h., except for the d.b.h. growth of crop trees where they entered first. Models containing the competition index of CRQ produced the highest r^2 values for all equations (table 4).

CONCLUSIONS

These competition measures are well correlated to tree growth and are the most valuable when evaluating the effects of thinning on potential crop trees. They should be included when analyzing the effect of stand improvement work or when the growth on quality trees is of primary concern. When total stand growth is of major interest, the overall improvement in predictive capabilities is probably not sufficient to offset the expense of obtaining tree coordinates, even though, statistically, these competition measures add significantly to the growth equations. Therefore, the major use of individual tree competition measures will be in thinning research or simulation, except in those cases where tree plot data are already available, or are easily obtainable such as in regularly spaced plantations.

Because three growing seasons is probably the minimum acceptable remeasurement time after treatment for northern hardwoods, these results should be verified with additional data or after additional growing time. Because the residual stand in this study was 93-percent sugar maple, with the rest red maple (less than 1-percent other species), the data were combined for analysis. The choice of competition measures depends on species, so stands of other forest types should also be analyzed.

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1992. **Calculating competition in thinned northern hardwoods**. Res. Pap. NC-306. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Forest Experiment Station. 10 p.

Describes four methods of calculating competition to individual trees and compares their effectiveness in explaining the 3-year growth response of northern hardwoods after various mechanized thinning practices.

KEY WORDS: Growth modeling, area potentially available, crown release, competition index.